

Leading Questions in an Extended Standard Model

W-Y. Pauchy Hwang^{1,1} and Tung-Mow Yan²

¹*Asia Pacific Organization/Center for Cosmology and Particle Astrophysics,
Institute of Astrophysics, Center for Theoretical Sciences,
and Department of Physics, National Taiwan University, Taipei 106, Taiwan*

²*Department of Physics, Cornell University, Ithaca, N.Y. 14850*

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Abstract

We would like to discuss the language to write an extended Standard Model - using renormalizable quantum field theory as the framework; to start with certain basic units together with a certain gauge group. Specifically we use the left-handed and right-handed spinors to form the basic units together with $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ as the gauge group. We could write down the extended Standard Model, though the details of the Higgs mechanism remains to be worked out. The same general quest appeared about forty years ago - the so-called "How to build up a model". It is timely to address the same question again especially since we could now put together "Dirac similarity principle" and "Higgs minimum hypothesis" as two additional working rules.

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1 Introduction

As time goes by, our confidence in what we are doing seems to be dwindling - so to look for "superstring", etc., for alternatives. The language which was built up during the last century, namely, renormalizable quantum field theory, may still be the language underlying the final Standard Model.

Usually in a textbook, the QCD chapter precedes the one on Glashow-Weinberg-Salam (GWS) electroweak theory. Nothing is wrong with it but the basic units (or the building blocks) are further divided into the left-handed and right-handed components. It would be nice (in helping us in thinking) if the framework is formulated all at once - in an extended Standard Model we could see everything consistent with one another. Then, the questions which we pose could have broader meanings and implications. Thus, this is what we wish to do.

We shall work with the Lie group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ as the gauge group. Thus, the basic units are made up from quarks (of six flavors, of three colors, and of the two helicities) and leptons (of three generations and of the two helicities), together with all originally massless gauge bosons and the somewhat hidden induced Higgs fields. In view of the search over the last forty years, we could assume "minimum Higgs hypothesis" as the working rule.

¹Correspondence Author; Email: wyhwang@phys.ntu.edu.tw; arXiv:xxxx (hep-ph, to be submitted); Not for publication.

If we look at the basic units as compared to the original particle, i.e. the electron, the starting basic units are all "point-like" Dirac particles. Dirac invented Dirac electrons eighty years ago and surprisingly enough these "point-like" Dirac particles are the basic units of the Standard Model. Thus, we call it "Dirac Similarity Principle" - a salute to Dirac; a triumph to mathematics. Our world could indeed be described by the proper mathematics. The proper mathematics may be the renormalizable quantum field theory, although our confidence in it sort of fluctuates in time.

There is no way to "prove" the above two working rules - "Dirac Similarity Principle" and "minimum Higgs hypothesis". It might be associated with the peculiar property of our Lorentz-invariant space-time. To use these two working rules, we could simplify tremendously the searches for the new extended Standard Models.

2 The Statement for the Extended Standard Model

So far, we have decided on the basic units - those left-handed and right-handed quarks and leptons; the gauge group is chosen to be $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$.

In the gauge sector, the lagrangian is fixed if the gauge group is given; only for a massive gauge theory, Higgs fields are called for and we postpone its discussions until we have spelled out the fermion sector.

For the fermion sector, the story is again fixed if the so-called "gauge-invariant derivative", i.e. D_μ in the kinetic-energy term $-\bar{\Psi}\gamma_\mu D_\mu\Psi$, is given for a given basic unit [1].

Thus, we have, for the up-type right-handed quarks u_R , c_R , and t_R ,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i \frac{2}{3} g' B_\mu, \quad (1)$$

and, for the rotated down-type right-handed quarks d'_R , s'_R , and b'_R ,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - i \left(-\frac{1}{3}\right) g' B_\mu. \quad (2)$$

On the other hand, we have, for the $SU_L(2)$ quark doublets,

$$D_\mu = \partial_\mu - ig_c \frac{\lambda^a}{2} G_\mu^a - ig \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu - i \frac{1}{6} g' B_\mu. \quad (3)$$

For the lepton side, we introduce the family triplet, $(\nu_\tau^R, \nu_\mu^R, \nu_e^R)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. We propose that neutrinos are only species seeing the family gauge sector. Or, we have, for $(\nu_\tau^R, \nu_\mu^R, \nu_e^R)$,

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a. \quad (4)$$

and, for the left-handed $SU_f(3)$ -triplet and $SU_L(2)$ -doublet $((\nu_\tau^L, \tau^L), (\nu_\mu^L, \mu^L), (\nu_e^L, e^L))$ (all columns),

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a - ig \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i \frac{1}{2} g' B_\mu. \quad (5)$$

The right-handed charged leptons are singlets under $SU_f(3)$, or the same as in the minimum Standard Model.

The Higgs mechanism in the minimal Standard Model remains the same. For the family gauge theory, we still hope to maintain [2] that all gauge bosons be massive, i.e. \geq a few TeV.

As slightly differing from the previous effort [2], we would like to write down the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Standard Model *all at once*, the mass term becomes

$$i\frac{\eta}{2}\bar{\Psi}^{R,T}(3,1) \times \Psi^L(3,2) \cdot \Phi(3,2) + \frac{\eta'}{2}\bar{\Psi}^R(\bar{3},1)\Psi^L(3,2)\Phi(1,2) + h.c., \quad (6)$$

where $\Psi(3,i)$ are the lepton multiplets just mentioned above (with the first number for $SU_f(3)$ and the second for $SU_L(2)$). The cross (curl) product is somewhat new [2], referring to the singlet combination of three triplets in $SU(3)$. The Higgs field $\Phi(3,2)$ is new in this effort, because it carries some nontrivial $SU_L(2)$ charge.

We add "T" explicitly to indicate the transpose in the $SU_f(3)$ space. By the "minimum Higgs hypothesis", the first coupling involves two spaces, the (3, 1) and (3, 2) internal spaces, which "should" be much smaller than, e.g., g , the interactions in the same space. By the same token, the second term involves three internal spaces; that is, it should be much further down compared to the first term. Note that the first term involves the singlet combination of three triplets - suitable for $SU(3)$; *not* an ordinary matrix operation.

On the two terms, the first one serves as the operator suitable for neutrino oscillations while the second one is the ordinary (diagonal) mass term, but maybe smaller than radiative mass corrections induced by familons (those from Eqs. (4), (5), etc.).

3 Lepton-flavor-violating Interaction

Neutrinos have masses, the tiny masses far below the range of the masses of the quarks and charged leptons. Neutrinos oscillate among themselves, giving rise to a lepton-flavor violation (LFV). There are other oscillation stories, such as the oscillation in the $K^0 - \bar{K}^0$ system, but there is a fundamental difference here - the $K^0 - \bar{K}^0$ system is composite while neutrinos are "point-like" Dirac particles. It is true that neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [3].

In fact, certain LFV processes such as $\mu \rightarrow e + \gamma$ [3] and $\mu + A \rightarrow A^* + e$ are closely related to the most cited picture of neutrino oscillations so far [3]. In recent publications by one [4] of us, it was pointed out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the new Higgs field(s).

In the other words, the first term in the last equation [Eq. (6)] can be used as the basis to analyze the various lepton-flavor-violating decays and reactions.

To illustrate the point further, we calculate the golden lepton-flavor-violating decay $\mu \rightarrow e + \gamma$ as the celebrated example. We show in Figures 1(a), 1(b), and 1(c) three leading basic Feynman diagrams. Here the conversion of ν_μ into ν_e is marked by a cross sign and it is a term from the off-diagonal interaction given above with the Higgs vacuum expectation

value u_0 . Here the Higgs masses are assumed to be very large, i.e., greater than a few TeV , as in $SU_f(3)$. The only small number (coupling) is η , consistent with the tiny masses of neutrinos.

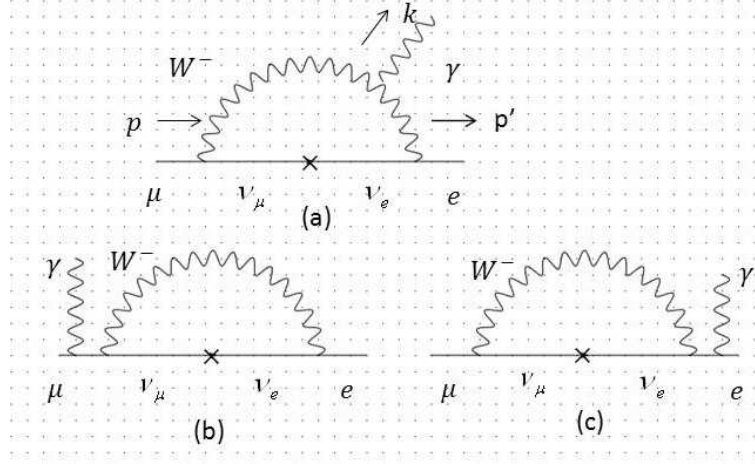


Figure 1: The leading diagrams for $\mu \rightarrow e + \gamma$.

Using Feynman rules from Wu and Hwang [1], we write, for Fig. 1(a),

$$\begin{aligned}
 & \frac{1}{(2\pi)^4} \int d^4q \cdot \bar{u}(p', s') \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_\lambda (1 + \gamma_5) \\
 & \cdot \frac{1}{i} \frac{m_2 - i\gamma \cdot q}{m_2^2 + q^2 - i\epsilon} \cdot i \cdot i\eta(-)u_0 \cdot \frac{1}{i} \frac{m_1 - i\gamma \cdot q}{m_1^2 + q^2 - i\epsilon} \\
 & \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_{\lambda'} (1 + \gamma_5) \cdot u(p, s) \\
 & \cdot \frac{1}{i} \frac{\delta_{\lambda'\mu}}{M_W^2 + (p-q)^2 - i\epsilon} \cdot \frac{\epsilon_\sigma(k)}{\sqrt{2k_0}} \cdot \Delta_{\sigma\mu\nu} \cdot \frac{1}{i} \frac{\delta_{\nu\lambda}}{M_W^2 + (p-q-k)^2 - i\epsilon},
 \end{aligned}$$

with $\Delta_{\sigma\mu\nu} = (-ie)\{\delta_{\mu\nu}(-k-p-q)_\sigma + \delta_{\nu\sigma}(p-q+p-q-k)_\mu + \delta_{\sigma\mu}(-p+q+k+k)_\nu\}$.

On the other hand, Feynman rules yield, for Fig. 1(b),

$$\begin{aligned}
 & \frac{1}{(2\pi)^4} \int d^4q \cdot \bar{u}(p', s') \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_\lambda (1 + \gamma_5) \\
 & \cdot \frac{1}{i} \frac{m_2 - i\gamma \cdot q}{m_2^2 + q^2 - i\epsilon} \cdot i \cdot i\eta(-)u_0 \cdot \frac{1}{i} \frac{m_1 - i\gamma \cdot q}{m_1^2 + q^2 - i\epsilon} \\
 & \cdot i \cdot \frac{1}{2\sqrt{2}} \frac{e}{\sin\theta_W} \cdot i\gamma_{\lambda'} (1 + \gamma_5) \cdot \\
 & \cdot \frac{1}{i} \frac{\delta_{\lambda\lambda'}}{M_W^2 + (p'-q)^2 - i\epsilon} \cdot \frac{1}{i} \frac{m_\mu - i\gamma \cdot p'}{m_\mu^2 + p'^2 - i\epsilon} \cdot i(-i)e \cdot \gamma \cdot \frac{\epsilon(k)}{\sqrt{2k_0}} \cdot u(p, s),
 \end{aligned}$$

and a similar result for Fig. 1(c).

The four-dimensional integrations can be carried out, via the dimensional integration formulae (e.g. Ch. 10, Wu/Hwang [1]), especially if we drop the small masses compared to the W-boson mass M_W in the denominator. In this way, we obtain

$$iT_a = \frac{G_F}{\sqrt{2}} \cdot \eta u_0 \cdot (m_1 + m_2) \cdot (-2i) \frac{e}{(4\pi)^2} \cdot \bar{u}(p', s') \frac{\gamma_5 \epsilon}{\sqrt{2} k_0} (1 + \gamma_5) u(p, s).$$

It is interesting to note that the wave-function renormalization, as shown by Figs. 1(b) and 1(c), yields

$$iT_{b+c} = \frac{G_F}{\sqrt{2}} \cdot \eta u_0 (m_1 + m_2) \cdot (+2i) \frac{e}{(4\pi)^2} \cdot \left\{ \frac{p'^2}{m_\mu^2 + p'^2} + \frac{p^2}{m_e^2 + p^2} \right\} \cdot \bar{u}(p', s') \frac{\gamma_5 \epsilon}{\sqrt{2} k_0} (1 + \gamma_5) u(p, s),$$

noting that $p^2 = -m_\mu^2$ and $p'^2 = -m_e^2$ would make the contribution from Figs. 1(b) and 1(c) to be of the opposite sign to that from Fig. 1(a).

It is interesting to note that the leading terms all cancel, a result of gauge invariance. We have computed some next-order terms but a complete result seems to be rather difficult to obtain.

In a normal treatment, one ignores the wave-function renormalization diagrams 1(b) and 1(c) in the treatment of the decays $\mu \rightarrow e + \gamma$, $\mu \rightarrow 3e$, and $\mu + A \rightarrow e + A^*$.

Comparing this to the dominant mode $\mu \rightarrow e \bar{\nu}_e \nu_\mu$ [1], we could obtain the branching ratio. Even though the decay rate for $\mu \rightarrow e + \gamma$ would be of the order $O(m_{\text{neutrino}}^4/M_W^4)$, which is extremely small. Note that the cancelation does not exist for $\mu \rightarrow e + e^+ + e^-$, nor for the conversion process $\mu^- + p \rightarrow e^- + p$. So, the rates would be expected to be much larger.

The off-diagonal mass matrix would be modified by the self-energy diagram since the neutrinos form a triplet under $SU_f(3)$. It is presumed that these self-energy diagrams, after the ultraviolet divergences get subtracted, lead to masses of the right order. If the off-diagonal mass matrix is diagonalized alone, the three roots would be two negative and one positive, adding up to zero. This seems like one ordering in the masses of neutrinos - one up and two downs.

Besides the golden decay $\mu \rightarrow e + \gamma$ (much too small) and neutrino oscillations (already observed), violation of the $\tau - \mu - e$ universality is also expected and might be observed. As the baryon asymmetry is sometime attributed to the lepton-antilepton asymmetry, the current scenario for neutrino oscillations [3] seems to be inadequate. If we take the hints from neutrinos rather seriously, there are so much to discover, even though the minimal Standard Model for the ordinary-matter world remains to be intact.

4 The Questions

Let us come back to look at the statement of the extended Standard Model. We choose the basic units at first and then the gauge group. The Higgs mechanism would be in the last step.

If that is the case, we have some difficulty in writing down the left-right model [5]. why? If we need to assign a certain left-handed or right-handed spinor into two basic units simultaneously, then the kinetic term appears twice - our language does not go; we believe that a lagrangian should have only one kinetic term.

So, our first question would be: Could the above rationale rule out the right-handed sector, since the simultaneous presence of the left-handed and right-handed basic units as $SU(2)$ doublets are excluded? Experimentally, we should check this point. As long as we could argue, we note that, as long as the left-handed and right-handed components are split in the basic units, parity has to be violated, either V-A or V+A.

In a slightly different context [6], It was proposed that we could work with two working rules: "Dirac similarity principle", based on eighty years of experience, and "minimum Higgs hypothesis", from the last forty years of experience. Using these two working rules, the extended model mentioned above becomes rather unique - so, it is so much easier to check it against the experiments.

The Model stated in the paper is yet to be completed, in view of the "minimum Higgs hypothesis". The Higgs mechanism in the previous $SU_f(3)$ family gauge theory is complete since the theory is treated *alone*. With $SU_f(3)$ and $SU_L(2)$ (3, 2) Higgs multiplet mentioned above plus one (3, 1) Higgs triplet, is it sufficient to do the Higgs-mechanism job - no "unwanted" massless particles? we would like to list this "mathematical" question as the second important question.

We would be curious about how the dark-matter world looks like, though it is difficult to verify experimentally. The first question would be: The dark-matter world, 25 % of the current Universe (in comparison, only 5 % in the ordinary matter), would clusterize to form the dark-matter galaxies, maybe even before the ordinary-matter galaxies. The dark-matter galaxies would then play the hosts of (visible) ordinary-matter galaxies, like our own galaxy, the Milky Way. Note that a dark-matter galaxy is by our definition a galaxy that does not possess any ordinary strong and electromagnetic interactions (with our visible ordinary-matter world). This fundamental question deserves some thoughts, for the structural formation of our Universe.

Of course, we should remind ourselves that, in our ordinary-matter world, those quarks can aggregate in no time, to hadrons, including nuclei, and the electrons serve to neutralize the charges also in no time. Then atoms, molecules, complex molecules, and so on. These serve as the seeds for the clusters, and then stars, and then galaxies, maybe in a time span of 1 *Gyr* (i.e., the age of our young Universe). The aggregation caused by strong and electromagnetic forces is fast enough to help giving rise to galaxies in a time span of 1 *Gyr*. On the other hand, the seeded clusterings might proceed with abundance of extra-heavy dark-matter particles such as familons and family Higgs, all greater than a few *TeV* and with relatively long lifetimes (owing to very limited decay channels). So, further simulations on galactic formation and evolution may yield clues on our problem.

So, we could put forward the third important question of this paper: What are the details of the dark-matter world?

Finally, coming back to the fronts of particle physics, neutrinos, especially the right-handed neutrinos, might couple to the dark-matter particles. Any further investigation along this direction would be of utmost importance. It may shed light on the nature of the dark-matter world.

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